Concrete-Representational-Abstract Approach for Expressions

Why Is This Strategy Useful?

To improve the opportunities for success in algebra, teachers and researchers developed an approach they call Concrete-Representation-Abstract to teach successfully concepts and operations involving fractions and basic algebra. The Concrete to Representational to Abstract sequence of instruction includes three stages of learning.

- **Concrete.** In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (e.g., red and yellow chips, cubes, base-ten blocks, pattern blocks, fraction bars, and geometric figures).

- **Representational.** In this stage, the teacher transforms the concrete model into a representational level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting.

- **Abstract.** At this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, −, ×, ÷) to indicate addition, multiplication, or division.

Specifically, this algebra model is designed to take students from reducing simple two-statement expressions to solving more complex equations. This model displays the conceptual components in its concrete and pictorial representation in a manner that prepares the student to succeed in more advanced algebra concepts. This approach to teaching algebra enables teachers to teach higher-level concepts in a manner that actively involves students, generalizes to complex equations, and adapts to individual learning styles. This strategy is appropriate for all secondary school algebra students and has been shown to be effective for students with disabilities.

**Description of Strategy**

When incorporating the CRA approach, teachers should use the following guidelines:

- Choose the math topic to be taught. Plan what is to be taught ahead of time. Sequence the lessons so they start basic and gradually introduce new topics.

- Review abstract steps to solve the problem. Ask, what is the desired math outcome of the group of lessons? Determine the procedural goal of the combination of math skills. List out the steps or procedures. Adjust the steps to eliminate notation or calculation tricks. Change or modify steps to create the most logical and sequential set of procedures.

- Match the abstract steps with an appropriate concrete manipulative. Initial understanding of content will be based on interactions with concrete objects, so be careful which ones you choose. The conceptual effectiveness of the manipulative object should be noted in accordance to the math skill being taught. Avoid concrete objects that only cover a few skills.

- Arrange concrete and representational lessons. Practice concrete manipulations. The same questions that you encounter you can be certain your students will as well. Practice how to mark pictorial representations that appear similar to concrete
manipulations. Make certain that your language throughout instruction matches the language required for the desired outcome.

- Teach each concrete, representational, and abstract lesson to student mastery (accuracy without hesitation). Model and guide students in their use of manipulative objects and pictorial representations. Teach students step by step gradually introducing mathematical vocabulary. Allow students to name or invent their stepwise procedures within instruction. Move from concrete to representational to abstract learning levels only after students show accuracy without hesitations in manipulations or drawings.

- Assess each level of learning according to stepwise procedures. Help students generalize learning through word problems and problem solving events. Incorporate word problems throughout a lesson to help show social relevance as to why a math skill is important to learn.

Research Evidence

At least one quasi-experimental design study supports the use of this strategy. This study included 358 middle school students. These students were matched into 34 pairs according to previous math course, standard test scores, age, ability, and grade level. Pretests of math achievement were obtained one week prior to implementation of the treatment. Posttest measures were obtained upon completion of the last day of treatment, and follow-up measures were obtained three weeks after that. Results indicated that the students who participated in CRA instruction outperformed the students who participated in traditional abstract instruction. They also committed fewer errors with negative numbers and with transformations of equations before solving for variables.

Sample Studies Supporting this Strategy


Thirty-four matched pairs of sixth- and seventh-grade students were selected from 358 participants in a comparison of an explicit concrete-to-representational-to-abstract (CRA) sequence of instruction with traditional instruction for teaching algebraic transformation equations. Each pair of students had been previously labeled with a specific learning disability or as at risk for difficulties in algebra. Students were matched according to achievement score, age, pretest score, and class performance. The same math teacher taught both members of each matched pair, but in different classes. All students were taught in inclusive settings under the instruction of a middle school mathematics teacher. Results indicated that students who learned how to solve algebra transformation equations through CRA outperformed peers receiving traditional instruction on both post-instruction and follow-up tests. Additionally, error pattern analysis indicated that students who used the CRA sequence of instruction performed fewer procedural errors when solving for variables.

Additional Resources


Concrete - Representational – Abstract Sequence of Instruction. [http://fcit.usf.edu/mathvids/strategies/cra.html](http://fcit.usf.edu/mathvids/strategies/cra.html)