Concrete-Representational-Abstract (CRA)

Why Is This Strategy Useful?

Students who struggle in mathematics often have trouble connecting visual and symbolic representations. Physical manipulatives, however, can make such connections explicit to students. CRA is a research-based intervention for mathematics instruction that can enhance the mathematics performance of students with learning disabilities. It is a three-part instructional strategy, with each part building on the previous instruction to promote student learning and retention, and to address conceptual knowledge. The CRA sequence aims to improve students’ conceptual understanding of fractions. Specifically, students should develop understanding of the following concepts:

- A fraction can represent equal parts of a whole.
- A fraction can represent a ratio between two quantities.
- A fraction can represent division.

CRA may be implemented at all grade levels individually, for small groups, or for the entire class. It can be used with children at the elementary or secondary level. It is especially well suited for students with mathematical disabilities.

Description of Strategy

The CRA instructional sequence consists of three stages: concrete, representational, and abstract. The instructional sequence begins with several lessons, which introduce the concept of fractions equivalence through the use of physical manipulatives, such as area models (e.g., commercially available fraction circles, rods, strips), number lines, rulers, scales, and set models (e.g., beans, candies). These lessons are then followed by several lessons focusing on representational drawings to represent fractions equivalence. Typically, three lessons each at the concrete and representational levels are needed. As a last step in this sequence, students practice solving abstract problems.

Research Evidence

At least one small randomized controlled trial demonstrated statistically significant positive effects of the CRA instructional sequence compared to an instructional sequence that included representational drawings and abstract problem solving only. This study was conducted with 48 students in Grades 6 through 8 in special education classrooms. Findings indicated that CRA was an effective and easily implementable strategy in middle school classroom settings with students who have mathematical disabilities.

Sample Studies Supporting This Strategy


This study investigated the effects of teaching middle school students with mathematics disabilities equivalent fraction concepts and procedures using the concrete-representational-abstract (CRA) instructional sequence or the representational-abstract (RA) instructional
sequence. Twenty-six students formed the CRA group, and 24 students formed the RA group. The two treatment groups received carefully sequenced instruction over 10 lessons. The only difference between the two treatment groups was that the CRA group used concrete manipulative devices for the first three lessons while the RA group used representational drawings. Analyses of the data indicated that students in both treatment groups improved overall in their understanding of fraction equivalency from pretest to posttest. On all achievement measures, students in the CRA group had overall higher mean scores than did students in the RA group. Implications for classroom instruction and suggestions for further research are discussed.

Sample Activity
(Source: Making and Investigating Fraction Strips. 
http://illuminations.nctm.org/LessonDetail.aspx?id=L540 )

Making and Investigating Fraction Strips

Students make and use a set of fraction strips to represent the length model, discover fraction relationships, and work with equivalent fractions.

Use of physical manipulatives to study fractions

To begin the lesson, give students six precut strips of paper in six different colors. Specify one color and have students hold up the strip of this color. Tell students that this strip will represent the whole. Have students write "one whole" on the fraction strip. The term "whole" is included in the labeling instead of "1" because it eliminates confusion between the one in fractions such as 1/2.

Next, ask students to pick a second strip, fold it, and cut it into two equal pieces. Ask them what they think each of these strips should be called (one-half or 1/2). Have students label their strips accordingly using both the word and the fractional representation. (Note: Students may prefer to highlight the fold marks, rather than physically cutting the individual fraction pieces.) Have students take out another strip, fold it, and cut it into four equal pieces. Ask them what they think each of these strips should be called (one-fourth or 1/4). Have students label their strips accordingly using both the word and the fractional representation. Repeat this process of folding, cutting, and naming strips for eighths, thirds, and sixths. Have students take out their "whole" and ask, "Which strip is 1/2 of the whole?" "Which strip is 1/4 of the whole?" Continue asking about 1/8, 1/3, and 1/6. Students should experiment with the strips until they are consistently arriving at the correct answer.

Have students work in pairs to line up their fraction strips and find as many relationships as they can. Have students record these relationships on paper. When they have finished, have them share the relationships they discovered. Record relationships on chart paper and discuss. Students will notice that one whole is the same as 2/2, 4/4, 8/8, 3/3, or 6/6. Another example includes the relationship between 1/2, 2/4, 4/8, and 3/6. Tell students that when fraction strips are the same length, they represent equivalent fractions.

Additional Resources

Concrete-Representational-Abstract instructional approach. 
http://www.k8accesscenter.org/training_resources/CRA_Instructional_Approach.asp